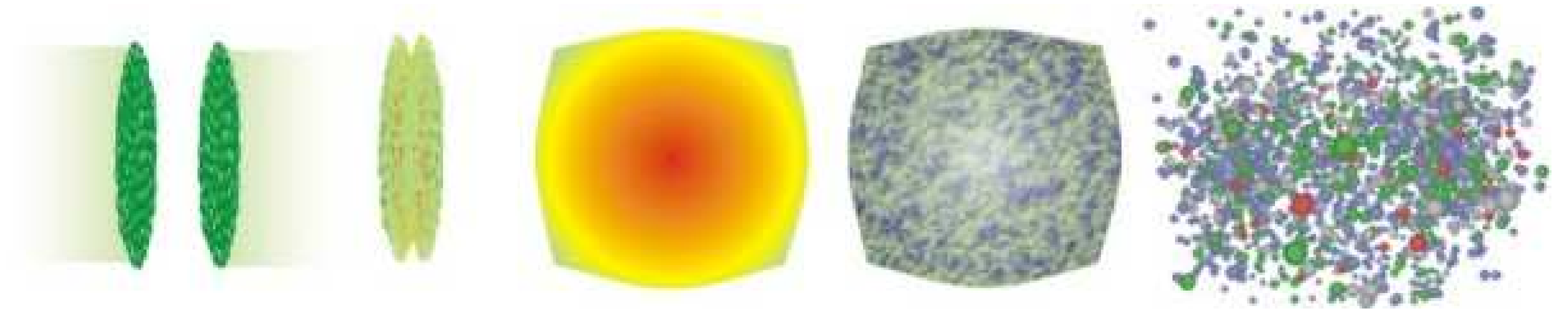
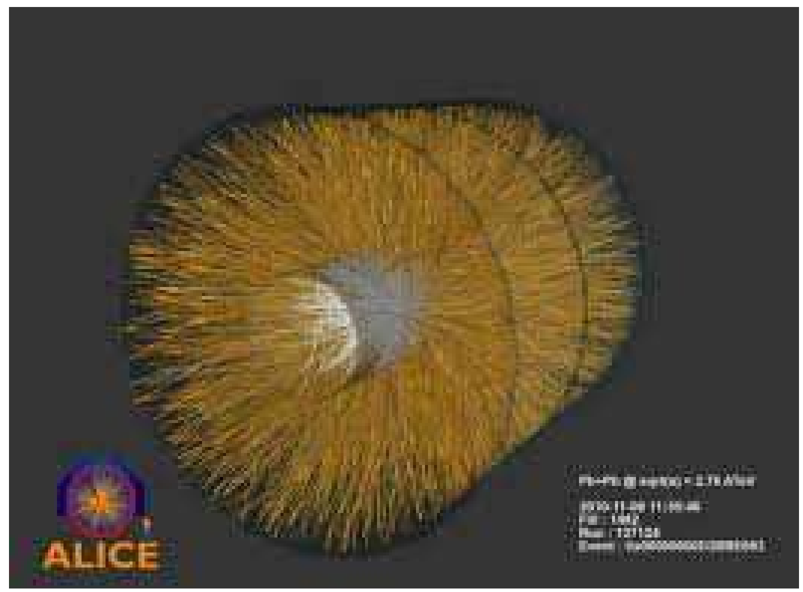
Transport in QCD: a Theorist's Perspective

Guy Moore, TU Darmstadt

- HIC, v_2 and flow: unnecessary review
- Hydrodynamics and derivative expansion
- Meaning of shear, bulk viscosity, quark diffusion etc
- Perturbation theory: strengths and weaknesses
- $N = 4\,\mathbf{SYM}$: strengths and weaknesses
- Lattice: strengths and weaknesses

LHC collides lead nuclei ($82p + 126n = 208$ nucleons)

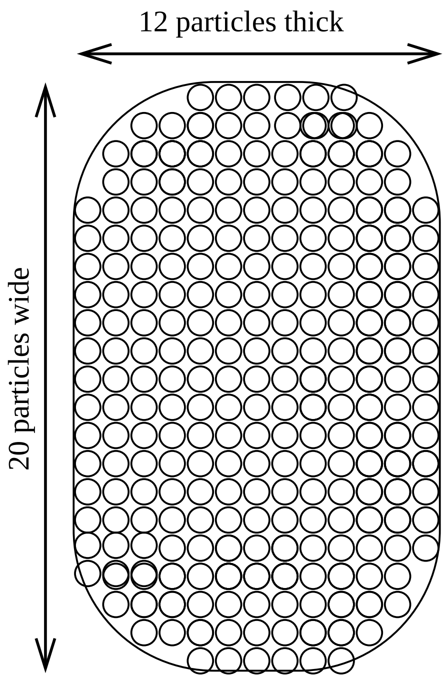




leading to 3200 charged,
> 1600 neutral particles
between $\theta = 40^\circ$ and
$\theta = 140^\circ$ $(-1 < \eta < 1)$

Each n, p gets "torn open," spilling out many $g, q, \bar{q}$ inside

Hot ball of 5000 excitations

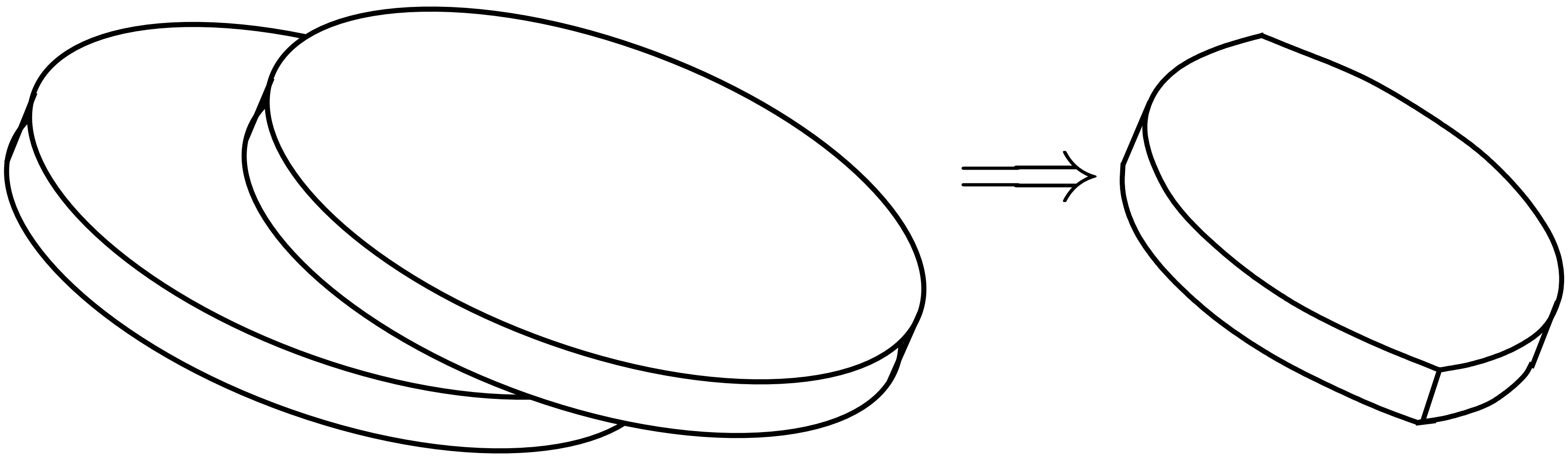


5000 excitations is around $20 \times 20 \times 12$ across. Enough to show collective or "fluid" behavior?

Hydrodynamics: Many "subsystems" big enough for *local* equilibration in each (Different regions with different $T, \vec{v}$,...). Not obvious but plausible

Testing for local equilibration

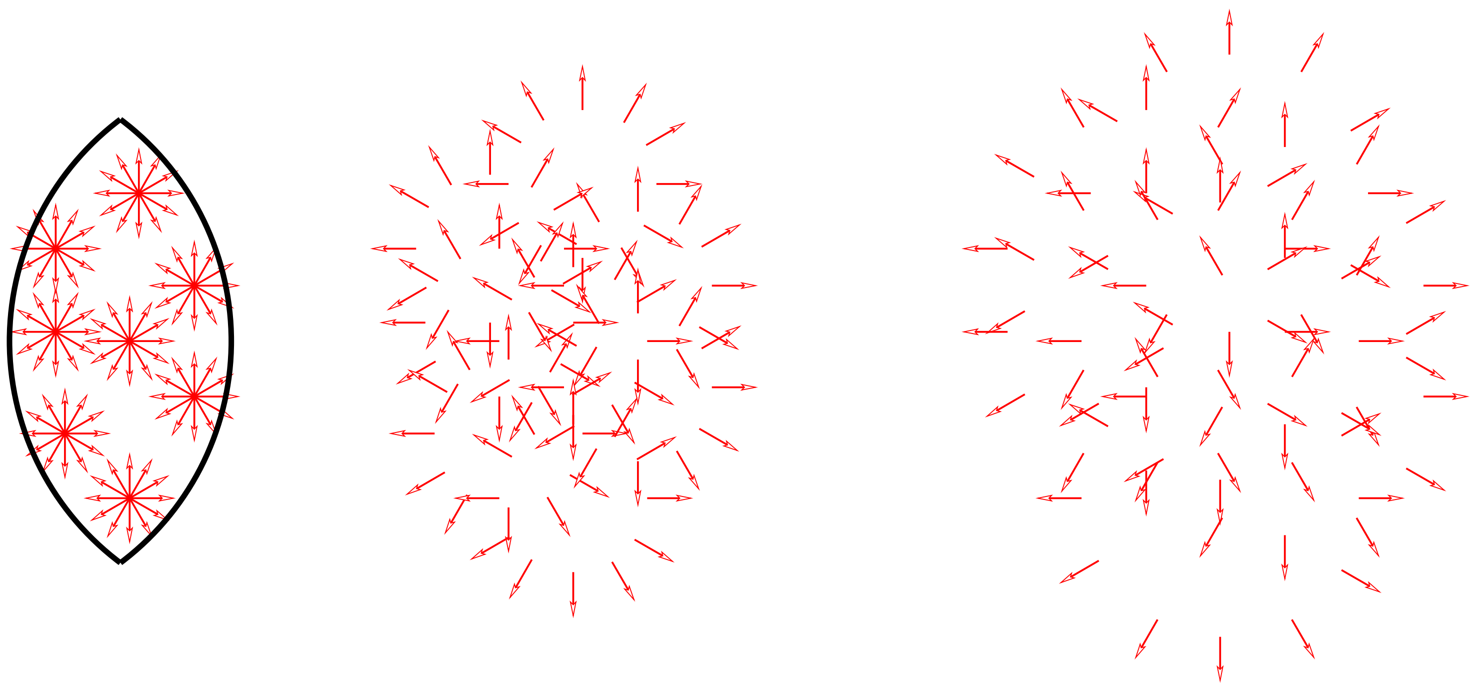
Nuclei generically strike off-center



leading to irregular shaped region of plasma

“Almond sliver” with long axis, short axis, and very short initial thickness along beam direction.

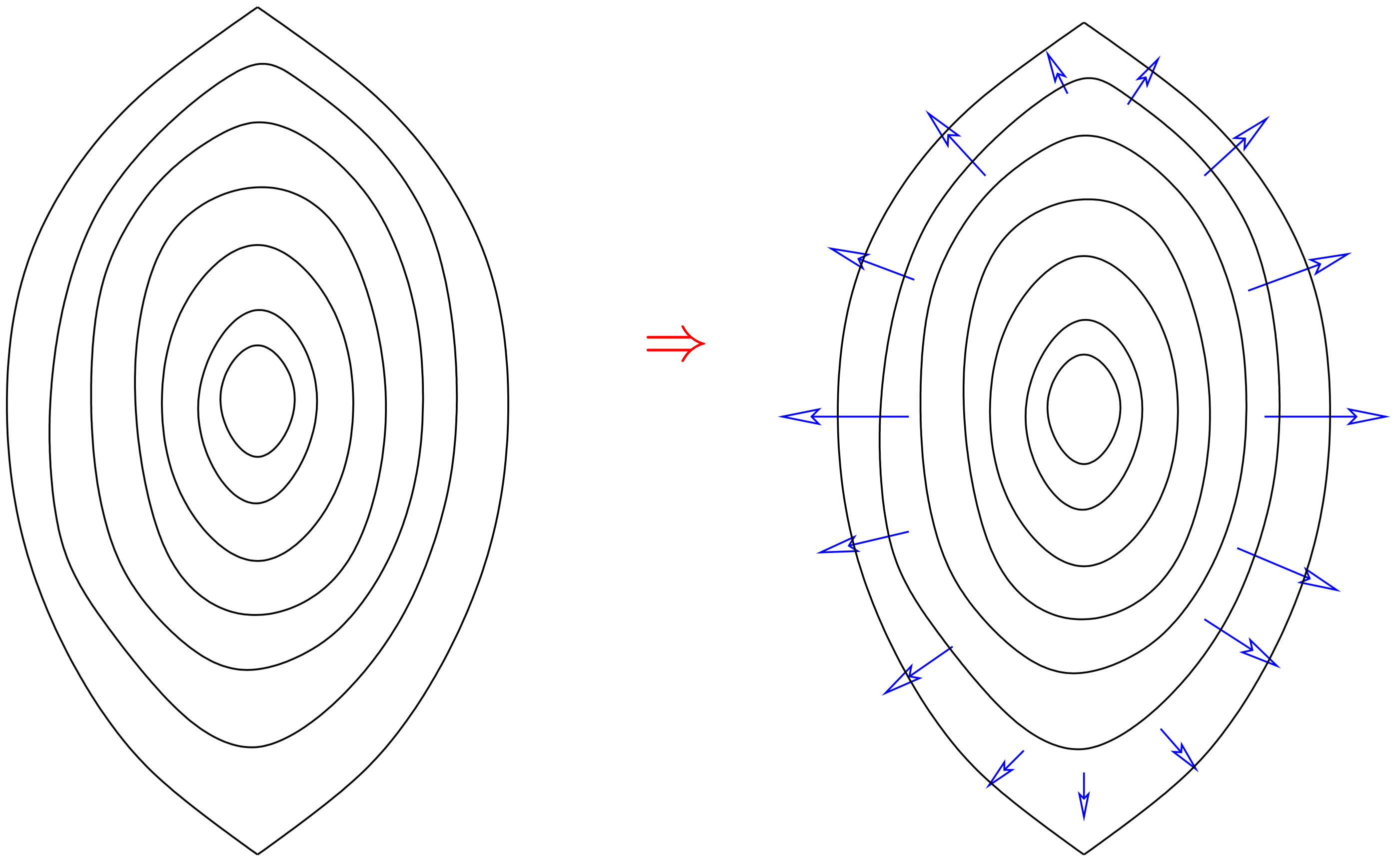
Behavior IF no re-interactions (transparency)



Just fly out and hit the detector.

Detector will see xy plane *isotropy*

local CM motions



Pressure contours   Expansion pattern

Anisotropy leads to anisotropic (local CM motion) flow.

Free particle propagation:

- System-average CM flow velocities $\langle v^2_{x,\mathrm{CM}}\rangle > \langle v^2_{y,\mathrm{CM}}\rangle$
- Must have local CM $\langle p^2_x\rangle < \langle p^2_y\rangle$ so total $\langle p^2_x\rangle = \langle p^2_y\rangle$

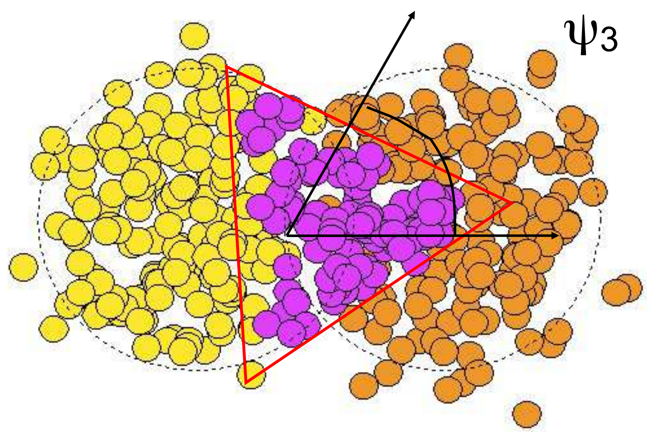
Efficient re-interaction:

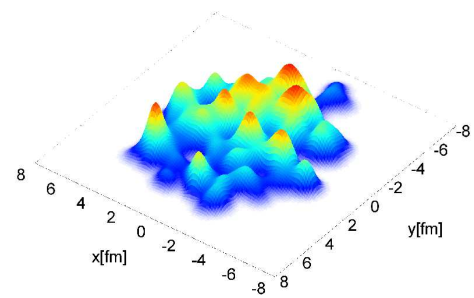
- System-average CM flow still has $\langle v^2_{x,\mathrm{CM}}\rangle > \langle v^2_{y,\mathrm{CM}}\rangle$
- system changes $locally$ towards $\langle T^{xx}_{\mathrm{local\ CM}}\rangle = \langle T^{yy}_{\mathrm{local\ CM}}\rangle$
- Adding these together, $\langle T^{xx}_{\mathrm{tot,labframe}}\rangle > \langle T^{yy}_{\mathrm{tot,labframe}}\rangle$

Net "Elliptic Flow" $v_2 \equiv \frac{p_x^2-p_y^2}{p_x^2+p_y^2}$ measures re-interaction

Higher harmonics

Nucleus is nucleons – clumpy substructure





.

Leads to fluctuating higher v_n moments. Amplitudes related. Higher n–shorter distance, more easily erased. Allows to measure what scales get "smeared" by streaming and which survive; sensitive probe of strength of re-interaction.

Perfect Rescattering: Ideal Hydrodynamics

System in local equilibrium. List all conserved quantities:

$$E, \quad \vec{p}, \quad Q_{\mathrm{el}}, \quad B \text{ [baryon number]}$$

Define local densities e, π, ρ, n, space varying. Local properties fixed by Equation of State:

$$-\Omega = P = P(e, \pi, \rho, n)$$ Only conserved quantities determine equilibrium.

Use Ω, thermodynamics to find conserved currents:

$$T^{\mu\nu}, \quad J^{\mu}_{Q}, \quad J^{\mu}_{\mathrm{B}}$$

Current conservation equations: 1 condition per unknown.

Ideal Hydrodynamics

Relativity: write $(e, \pi) = \frac{\varepsilon}{\sqrt{1-v^2/c^2}}(u^0, \vec{u})$: u^μ flow 4-vector,
$u^0 = \frac{1}{\sqrt{1-v^2/c^2}}$, $\vec{u} = \frac{\vec{v}/c}{\sqrt{1-v^2/c^2}}$

At rest, $T_{00} = \varepsilon$ and $T_{ij} = P\delta_{ij}$. Relativity:

$$T^{\mu\nu} = (\varepsilon + P)u^\mu u^\nu + P g^{\mu\nu}$$

with $g^{\mu\nu}$ the metric tensor. Conservation:

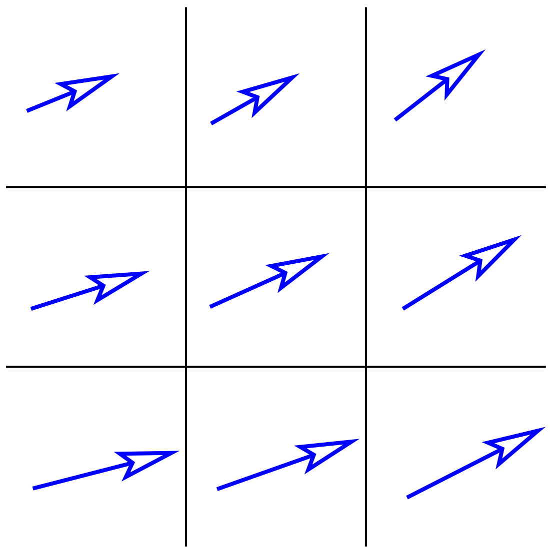
$$\nabla_\mu T^{\mu\nu} = 0$$

small $\vec{v}/c$: turns into usual Euler fluid eq.

Nonideal Hydro

Each region feels information about neighboring regions diffusing across its boundary.

$\vec{v}$ nonuniformity means nonvanishing $\nabla_i v_j$ which will influence center region (diffusion of information)

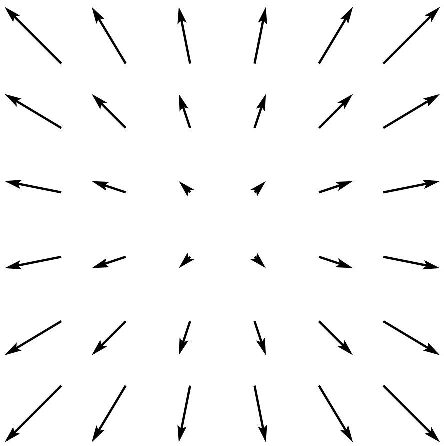


Decompose: scalar, antisymm, traceless symm tensor

$$\nabla_i v_j = \frac{\delta_{ij}}{3}\nabla \cdot v + \frac{1}{2}(\nabla_i v_j - \nabla_j v_i) + \frac{1}{2}\left(\nabla_i v_j + \nabla_j v_i - \frac{2\delta_{ij}}{3}\nabla \cdot v\right)$$

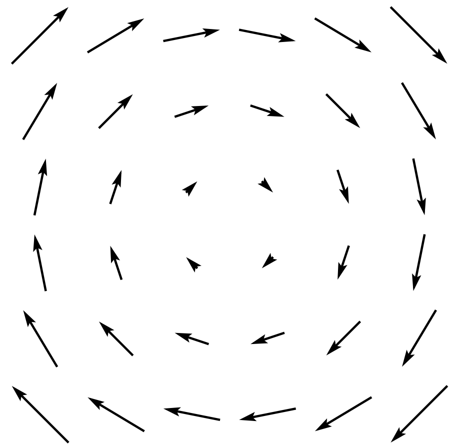
What each tensor piece means

$\delta_{ij}\nabla\cdot v$



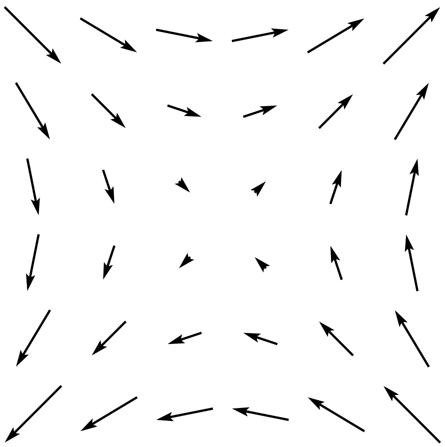
Divergence

$\nabla_i v_j - \nabla_j v_i$



Vorticity

$\nabla_i v_j + \nabla_j v_i - ..$



Shear flow

scalar divergence can change scalar pressure $P \Rightarrow P_{\text{equil.}} - \zeta\nabla\cdot v$

symm. tensor shear flow can change symm. tensor stress tensor

$T_{ij} \Rightarrow T_{ij,\text{equil.}} - \eta(\nabla_i v_j + \nabla_j v_i - ..)$

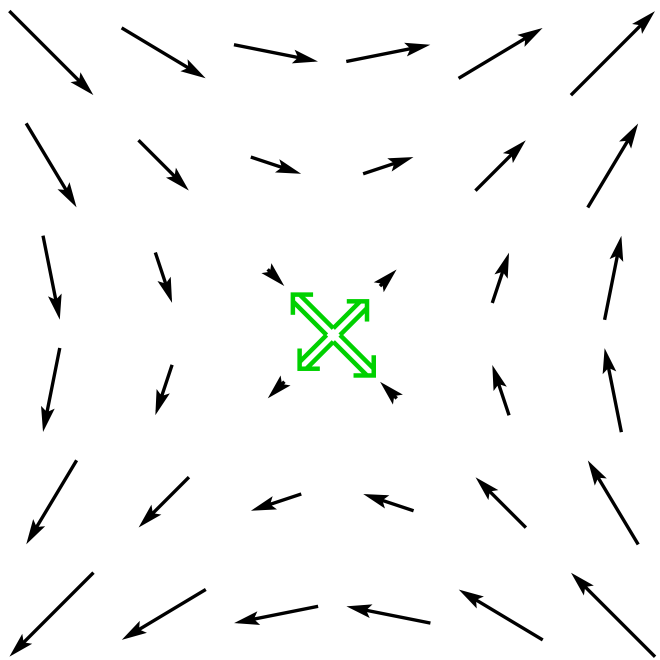
pseudovector vorticity cannot change either

Nonideal Hydro: Viscosity

Write $T^{\mu\nu}$ to first order in gradients:

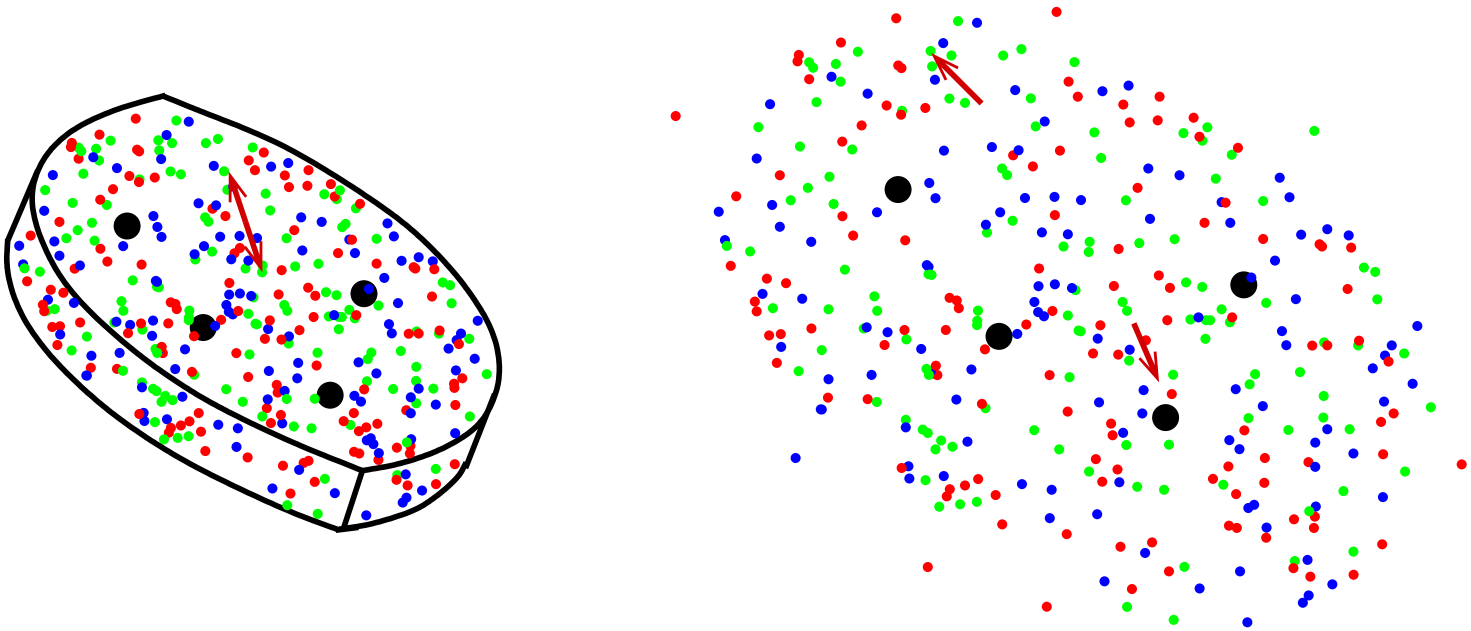
$$T^{\mu\nu} = (\varepsilon{+}P)u^\mu u^\nu + Pg^{\mu\nu} - \eta\sigma^{\mu\nu} - \zeta\nabla\cdot u(g^{\mu\nu} + u^\mu u^\nu)$$

Shear: anisotropy/ flow inhomogeneity



Bulk: pressure drop/ flow divergence.

Other transport coefficients



Collision makes **Heavy Quarks** and **Hard Partons**.

Each is buffeted by medium as emerges.

Quarks: slow moving, $v \simeq 0$. Partons: $v = 1$.

Momentum diffusion for $v = 0$ and $v = 1$ (?)

General approach: Kubo relation

Relativity: I can create shear flow $\nabla_i u_j$ by shearing my geometry $\partial_t h_{ij}$!
Instantaneous effect: $T^{ij} = Ph^{ij}$, P the pressure
Total effect: $T^{ij} \sim P\tau\partial_t h^{ij}$; τ some averaged relax. rate

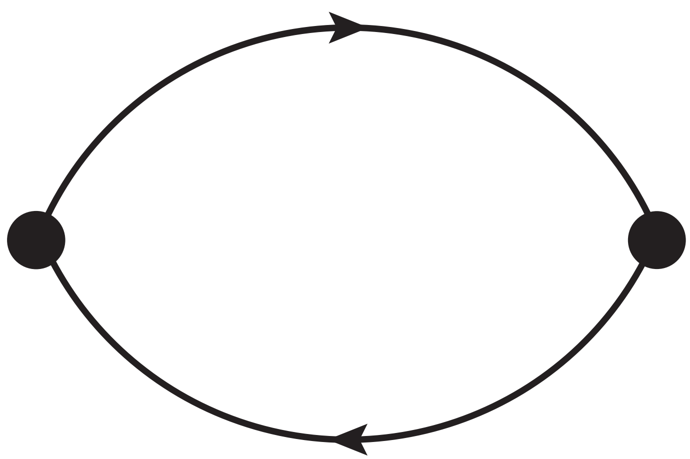
Kubo: thermal fluctuations in T^{ij}. Find τ by seeing how they relax. Result:

$$\eta = \frac{1}{2T} \int d^4x \langle T^{xy}(x) T^{xy}(0) \rangle$$

Starting point of most calculations.

Perturbative treatment

High temperature (hopefully achieved?): α_s small?

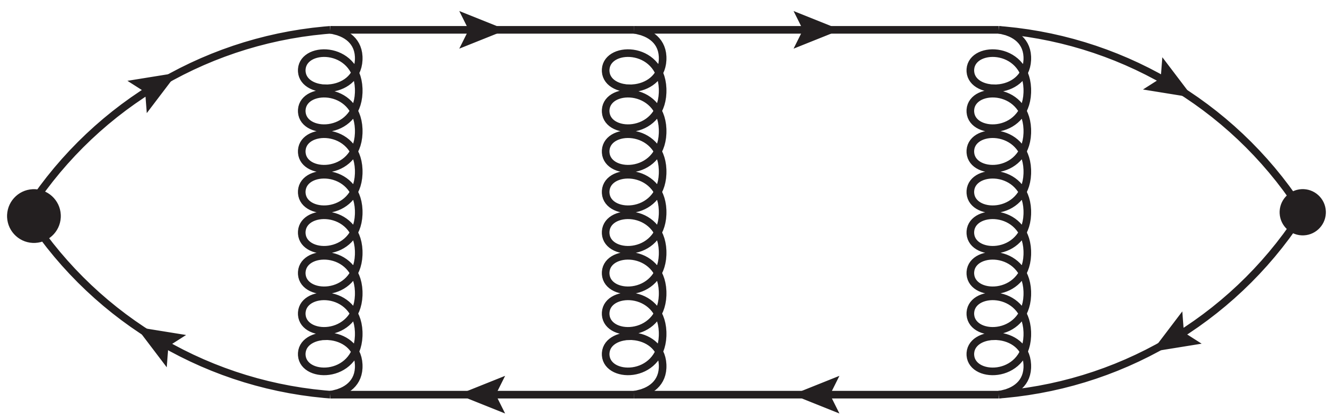


$\langle T^{xy} T^{xy} \rangle$ 2-propagators
Each $1/(p^2 + \Pi(p))$,
$\Pi \sim g^2 T^2$ with Re, Im
parts.

Dominant: $p^2 \simeq 0$, result $\propto 1/\mathrm{Im}\Pi$. Quasiparticles!
Implies $\eta \sim T^3/g^2$ – inverse powers of g.

Ladders!

Add rungs: More on-shell pairs

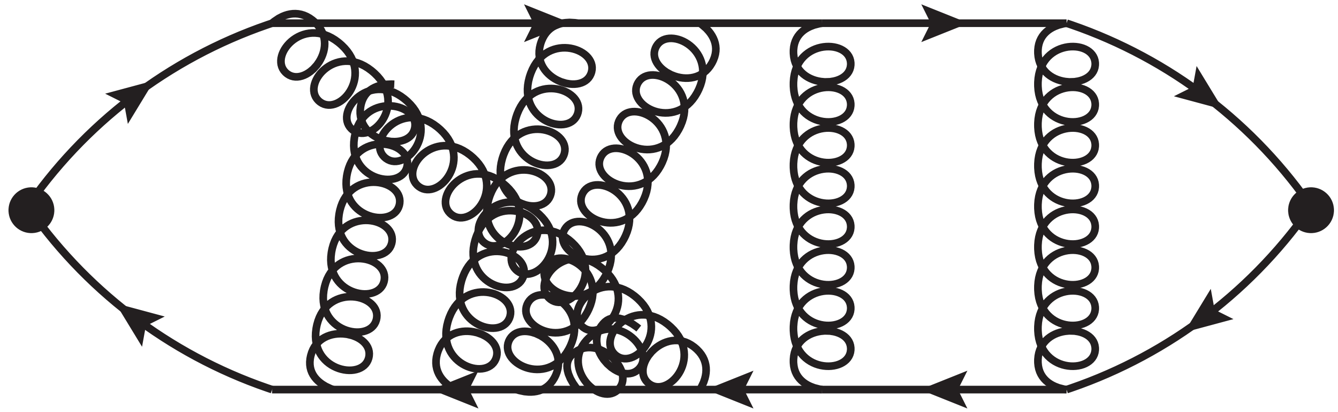


n-rung ladder $\simeq (1-g^2)\times$ (n-1)-rung ladder.

Need $\mathcal{O}(1/g^2)$ diagrams: $\eta \sim T^3/g^4$

More ladders!

Collinear physics requires including still-uglier graphs



Resummation a challenge but solved in 2002-3 Arnold GM Yaffe

hep-ph/0302165

Similar story for bulk viscosity Arnold Dogan Moore hep-ph/0608012

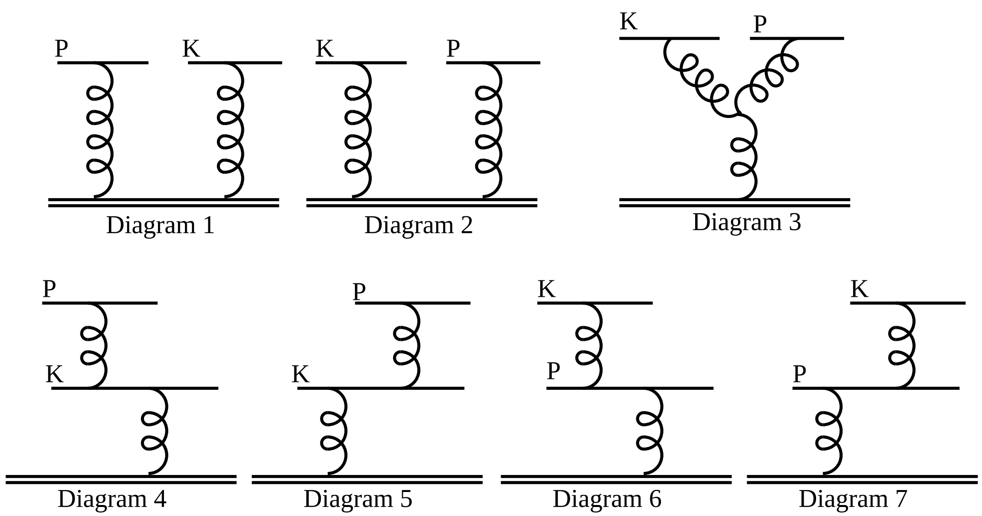
All this to get leading-order behavior!!

Next-to-leading order?

Perturbative series can surprise you (ϵ-expansion)

Know convergence only by finding a few terms.

Discussion so far was just for leading-order!

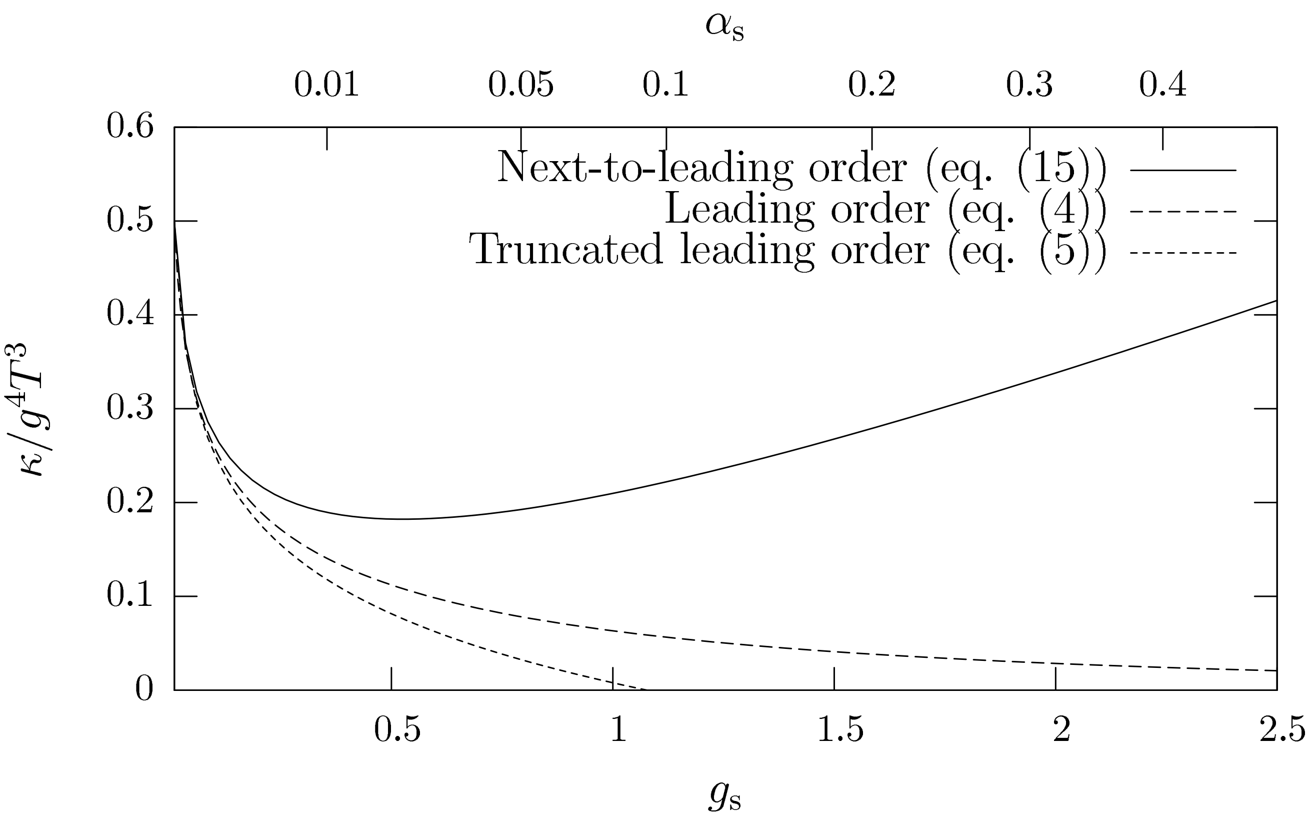


Complex interference effects arise at NLO.

Resummed for heavy-quarks in 2007, shear in 2016

Heavy Quark Diffusion at NLO

Diffusion at LO and NLO as function of α_s:

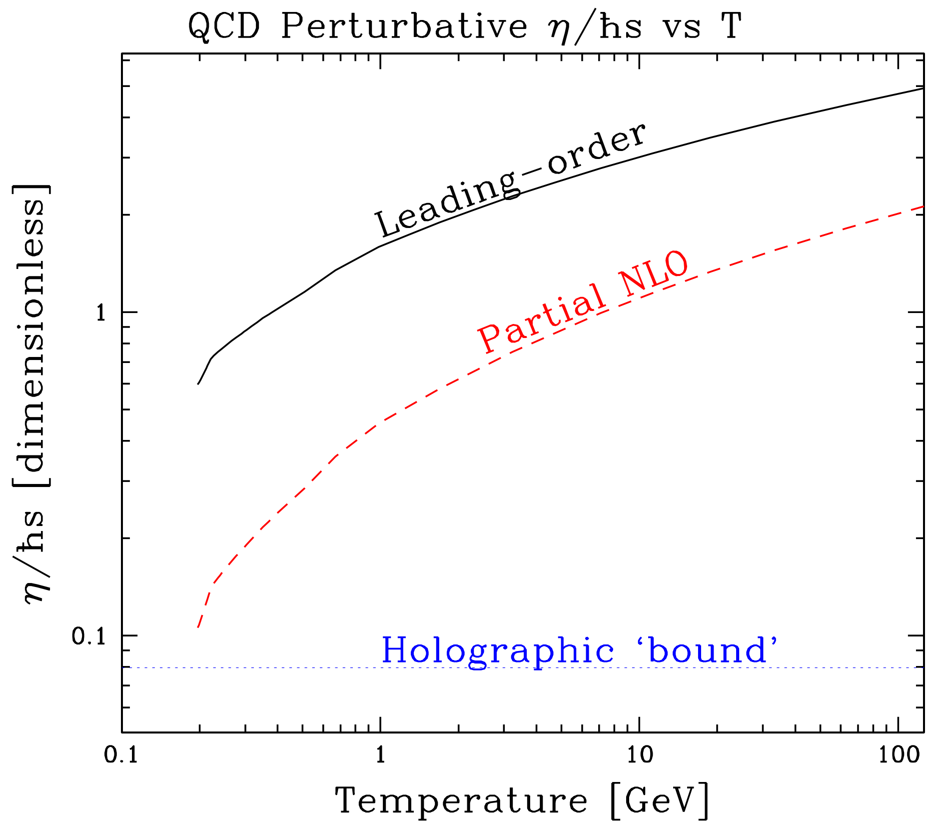


Perturbative expansion a total disaster! Caron-Huot GM arXiv:0801.2173

Shear viscosity at NLO

Recently completed (unpublished) almost-NLO treatment

Express as dimensionless ratio η/s: LO vs NLO



Very large downward NLO corrections.

Perturbation Theory: Summary

Positives:

- Really solving QCD
- Familiar methodology, physical interpretation

Negatives:

- Physically relevant T-range far out of reach
- Not much more than informed estimate
- Probably QCD at $T \sim 200$MeV is not Quasiparticles.

SYM approach

$\mathcal{N} = 4$SYM is a theory similar to QCD:

- Gauge group $SU(N)$ (though keep $N \gg 1$)
- 4 Weyl $\simeq 2$ Dirac fermions (though in adjoint rep)
- 6 real adjoint scalars (That's new)
- Certain Yukawa and scalar self-couplings (Also new)

So not quite QCD, but at least $SU(N)$ with fermions

Solving the theory

Holographic method: theory $=$ Type-IIB strings. Simplify:

- many colors $N \gg 1 \rightarrow$ supergravity limit
- strong coupling $g^2 N \equiv \lambda \gg 1 \rightarrow$ classical limit

Solve classical gravity in $\mathrm{AdS}^5 \times S^5$; SYM is boundary.
$g^2 N \gg 1$ possible because theory is conformal.
No confinement or asymptotic freedom.

Results (not mine)

"specific" shear viscosity takes universal value

$$\frac{\eta}{\hbar s} = \frac{1}{4\pi}$$

Kovtun Son Starinets hep-th/0405231 (Water at STP has $\eta/\hbar s = 33$)

First corrections: $\mathcal{O}(\lambda^{-3/2})$,

100% for $\lambda = 8$ Buchel Liu Starinets hep-th/0406264

Heavy quark diffusion is

$$D = \frac{2}{\pi T\sqrt{\lambda}}$$

Casalderrey-Solana Teaney hep-ph/0605199

Explicit leftover dependence on λ.

SYM: Summary

Positives:

- Solvable at strong coupling!
- Shows η/s can be small, Quasiparticles need not exist

Negatives:

- Wrong theory. Uncontrolled systematic error
- "realistic" coupling $\lambda \sim 10$ not under theoretical control

Lattice

Lattice QCD gives method to take (Euclidean) path integral

$$Z = \int \mathcal{D}A_\mu \exp\left(-\int_0^\beta \int d^3x \frac{1}{2g^2} \,\mathrm{Tr}\, G_{\mu\nu}G^{\mu\nu}\right)$$

or really, correlation functions

$$\langle T(y)T(0)\rangle = \frac{1}{Z}\int \mathcal{D}A_\mu \exp\left(\ldots\right) T(y)T(0)$$

But note, $y, 0$ at same time or differ by *imaginary* time.

What good does that do?

Where lattice is great

Zero-temperature masses:

$$\langle \hat{\pi}(x)\hat{\pi}(0)\rangle = Ce^{-xm_\pi}$$

exponential falloff of interpolating-operator correlator.

Thermodynamics:

$$\langle T^\mu_\mu \rangle \quad \text{or} \quad \langle T_{ii} \rangle$$

allow to reconstruct equation of state.

Nowadays, physical m_q, small statistical, syst. errors.

Transport coefficients?

Shear viscosity defined as:

$$\eta = \frac{1}{2T} \int d^3x dt \langle T_{xy}(x,t) T_{xy}(0,0) \rangle$$

What I can compute is

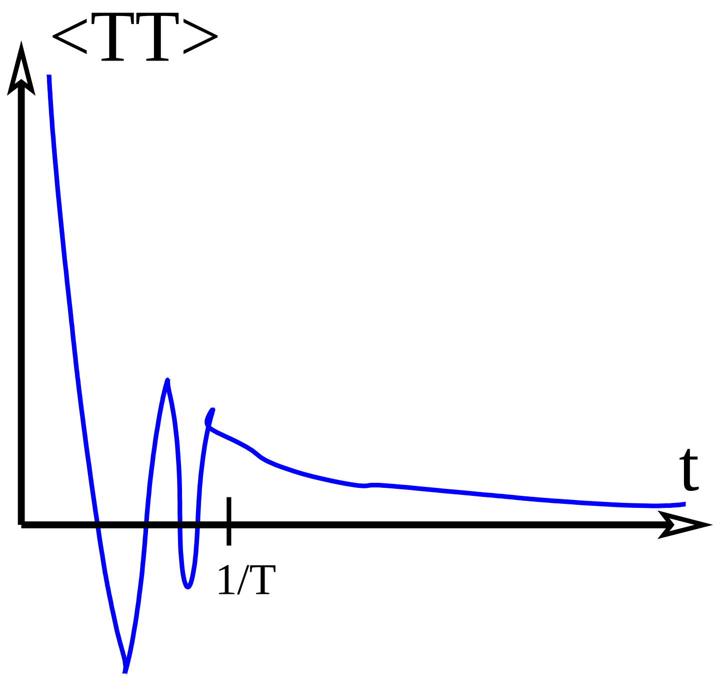
$$G(\tau) = \frac{1}{2T} \int d^3x \langle T_{xy}(x,i\tau) T_{xy}(0,0) \rangle \qquad \text{for} \quad \tau \in [0, 1/T] \, .$$

Related but not the same.

To get from one to other, use analytic structure of $\langle TT \rangle$

Spectral function

Consider time-structure of $\int d^3x \langle T(x,t) T(0,0) \rangle$:



Small-t: vacuum stuff.

Large-t: thermal decay.

Thermal part relevant.

Capture structure with spectral function

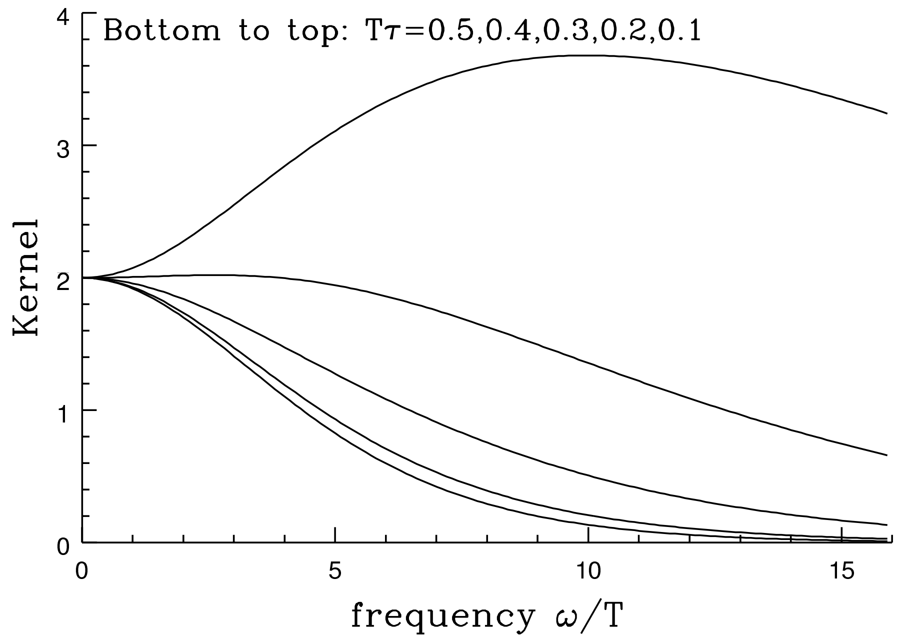
$\rho(\omega) = \int d^3x \int dt e^{i\omega t} \langle \Big[T(x,t) \, , \, T(0,0) \Big]$

Spectral function vs $G(\tau)$

Analytic relation between $t, \tau = it$ gives:

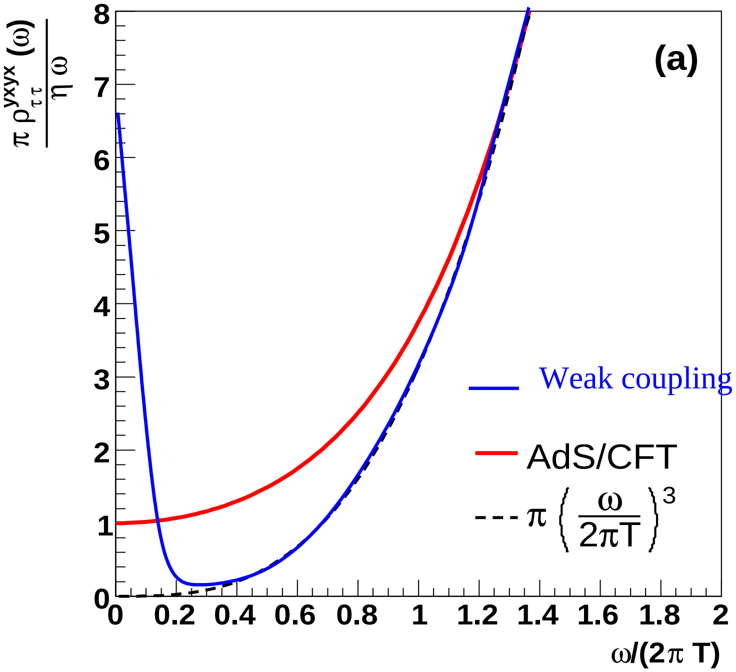
$$G(\tau) = \int \frac{d\omega}{2\pi} \frac{\rho(\omega)}{\omega} K(\omega,\tau)\,, \qquad K(\omega,\tau) = \frac{\omega \cosh(\omega(\tau - 1/2T))}{\sinh(\omega/2T)}$$

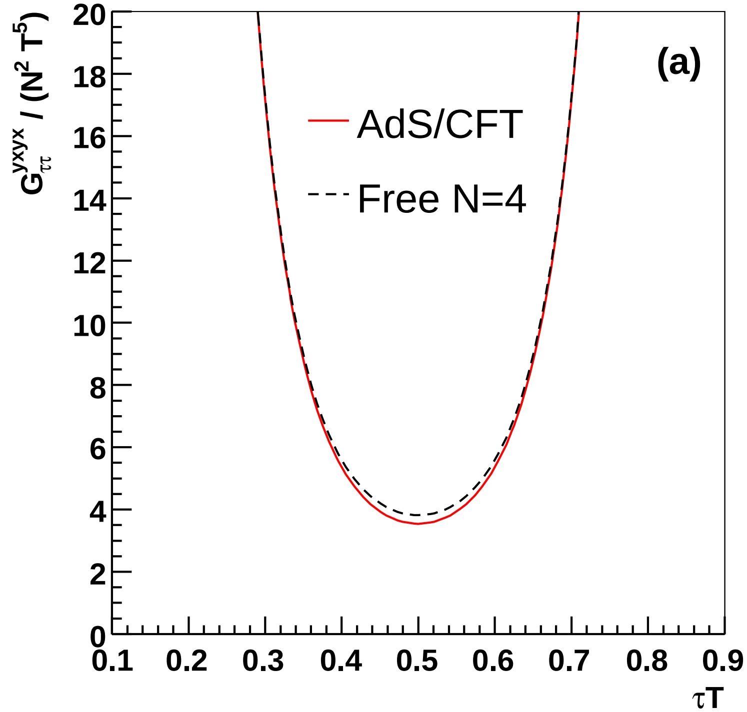
Here is $K(\omega,\tau)$
as function of ω
for several τ.



The danger:

Weak-coupling vs Strong: ρ left, $G(\tau)$ right





Huge changes near $\omega = 0$ (viscosity)

from tiny changes in $G(\tau)$. Teaney hep-ph/0602044

Reconstructing ρ

General info is OK, but structure near $\omega = 0$ (shear!) very hard to get right.
Most fitting methods effectively assume there is not a sharp structure near $\omega = 0$.

Doomed to find small η/s.

More analytical info about large-ω would help.
Generally reconstruction is fraught.

Lattice: summary

Positive:

- Right theory
- Nonperturbative info at strong coupling

Negative:

- Most results in quenched approx
  (pure-glue. Wrong theory)
- Systematic error to reconstruct η is **large**
- Systematic error may be *under-reported.*

Ways forward for theory?

Getting η/s for true QCD at interesting temperature will be tough. Not yet a clear approach with small errors.

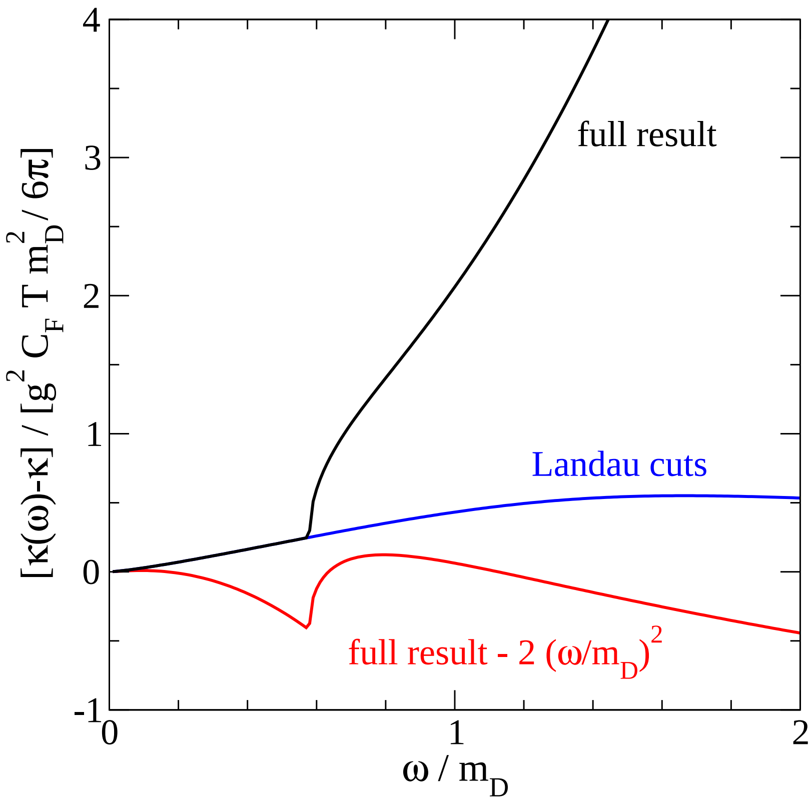
But there are other questions to ask!

- Heavy quark diffusion: different lattice approach may have much better systematic issues!
- High-energy quark momentum diffusion: dimensional reduction approach

Heavy quarks from the Lattice

Integrate out heavy quark: momentum diffusion from Force-force correlator, $g^2 \int dt \langle \vec{E}(0,t)\vec{E}(0,0)\rangle$ Caron-Huot Laine GM arXiv:0901.1195

Continuation to $E(\tau)E(0)$ on Polyakov loop. Weak coupling predicts **no peak** in $\rho(\omega)$. So continuation much safer.



Hard-particle transverse momentum exchange

Jet modification probably controlled by hard-particle $q_\perp$ exchange with medium:

$$\frac{d\Gamma}{dt} \equiv \int \frac{d^2 q_\perp}{(2\pi)^2} C(q_\perp)$$

$C(q_\perp)$ chance per-$d^2 q_\perp$-per-t to exchange $q_\perp$ momentum with medium.

Perturbative estimates, as usual.
NLO corrections large, as usual.

Can we get this from the lattice?

$C(q_\perp)$ from the lattice

If $T \gg T_c$ (maybe 2-3$\times$??), Dimensional Reduction works
Short-distance physics is perturbative
Long-distance physics nonperturbative *but*
described by 3D EFT: EQCD. Lattice treatment *easy*

Result Caron-Huot arXiv:0811.1603: $C(q_\perp)$ in terms of a "twisted" Wilson loop in EQCD.

Lattice implementation easy but fails without lattice improvement, now worked out D'Onofrio Kurkela GM arXiv:1401.7951

Numerical results to follow . . . .

Conclusions

- We need theory estimates of transport coefficients!
- Viscosity from Perturbation Theory: poor convergence
- Viscosity from SYM: big systematics (wrong theory)
- Viscosity from Lattice: big systematics (continuation)
- Heavy quarks from Lattice: looks hopeful!
- $C(q_\perp)$ from lattice: also hopeful!